Plasma Current, Position and Shape Control
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Outline

Introduction

Plasma Magnetic Modeling

Plasma Vertical Stabilization Problem

Plasma Shape Control Problem

Plasma Current Control problem

Plasma Position and Shape Control at JET

- eXtreme Shape Controller
- Vertical Stabilization System

Plasma Position and Shape Control at ITER

Motivation

- Plasma control is the crucial issue to be addressed in order to achieve the high performances envisaged for future tokamak devices
- High performance in tokamaks is achieved by plasmas with elongated poloidal cross section, which are vertically unstable
- Plasma magnetic **axisymmetric** control (shape and position) is an essential feature of all tokamaks
- If high performance and robustness are required, then a model-based design approach is needed

This lecture

1. focuses on plasma shape control and the vertical stabilization problems
2. presents the eXtreme Shape Controller (XSC) and the new Vertical Stabilization System (VS) recently deployed at the JET tokamak
3. briefly introduces a plasma position and shape control approach proposed for the ITER tokamak

Main assumptions

1. The plasma/circuits system is axisymmetric
2. The inertial effects can be neglected at the time scale of interest, since plasma mass density is low
3. The magnetic permeability μ is homogeneous, and equal to μ_0 everywhere

Mass vs Massless plasma

Recently, it has been proven that neglecting plasma mass may lead to erroneous conclusion on closed-loop stability.

M. L. Walker and D. A. Humphreys,
A multivariable analysis of the plasma vertical instability in tokamaks
Proc. 45th Conf. Decision Control, San Diego, CA, Dec. 2006, pp. 2213–2219

Plasma model

The *input variables* are:

- The voltage applied to the active coils v
- The plasma current I_p
- The poloidal beta β_p
- The internal inductance l_i

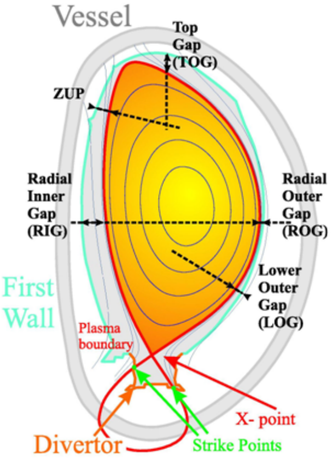
I_p , β_p and l_i

I_p , β_p and l_i are used to specify the current density distribution inside the plasma region.

Model outputs

Different model outputs can be chosen:

- fluxes and fields where the magnetic sensors are located
- currents in the active and passive circuits
- plasma radial and vertical position (1st and 2nd moment of the plasma current density)
- geometrical descriptors describing the plasma shape (gaps, x-point and strike points positions)



Lumped parameters approximation

By using finite-elements methods, **nonlinear** lumped parameters approximation of the PDEs model is obtained

$$\frac{\mathrm{d}}{\mathrm{dt}}\left[\mathcal{M}\big(\mathbf{y}(t), \beta_p(t), l_i(t)\big)\mathbf{I}(t)\right] + \mathbf{R}\mathbf{I}(t) = \mathbf{U}(t)\,,$$

$$\mathbf{y}(t) = \mathcal{Y}\big(\mathbf{I}(t), \beta_p(t), l_i(t)\big)\,.$$

where:

- $\mathbf{y}(t)$ are the output to be controlled
- $\mathbf{I}(t) = \left[\mathbf{I}_{PF}^T(t)\ \mathbf{I}_e^T(t)\ I_p(t)\right]^T$ is the currents vector, which includes the currents in the active coils $\mathbf{I}_{PF}(t)$, the eddy currents in the passive structures $\mathbf{I}_e(t)$, and the plasma current $I_p(t)$
- $\mathbf{U}(t) = \left[\mathbf{U}_{PF}^T(t)\ \mathbf{0}^T\ 0\right]^T$ is the input voltages vector
- $\mathcal{M}(\cdot)$ is the mutual inductance nonlinear function
- $\mathbf{R}$ is the resistance matrix
- $\mathcal{Y}(\cdot)$ is the output nonlinear function

Plasma linearized model

Starting from the nonlinear lumped parameters model, the following plasma linearized state space model can be easily obtained:

$$\delta\dot{\mathbf{x}}(t) = \mathbf{A}\delta\mathbf{x}(t) + \mathbf{B}\delta\mathbf{u}(t) + \mathbf{E}\delta\dot{\mathbf{w}}(t), \quad (1)$$

$$\delta\mathbf{y}(t) = \mathbf{C}\ \delta\mathbf{I}_{PF}(t) + \mathbf{F}\delta\mathbf{w}(t), \quad (2)$$

where:

- **A**, **B**, **E**, **C** and **F** are the model matrices
- $\delta\mathbf{x}(t) = \left[\delta\mathbf{I}_{PF}^T(t)\ \delta\mathbf{I}_e^T(t)\ \delta I_p(t)\right]^T$ is the state space vector
- $\delta\mathbf{u}(t) = \left[\delta\mathbf{U}_{PF}^T(t)\ \mathbf{0}^T\ 0\right]^T$ are the input voltages variations
- $\delta\mathbf{w}(t) = \left[\delta\beta_p(t)\ \delta l_i(t)\right]^T$ are the β_p and l_i variations
- $\delta\mathbf{y}(t)$ are the output variations

The model (1)–(2) relates the variations of the PF currents to the variations of the outputs around a given equilibrium

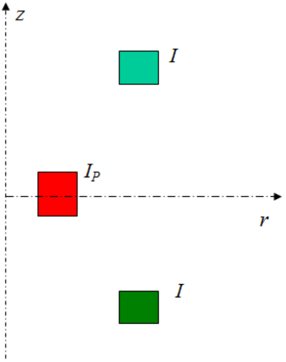
Vertical Stabilization Problem

Objectives

- Vertically stabilize elongated plasmas in order to avoid disruptions
- Counteract the effect of disturbances (ELMs, fast disturbances modelled as VDEs,...)
- It does not control vertical position but it *simply* stabilizes the plasma
- The VS is the essential magnetic control system!

Simplified filamentary model

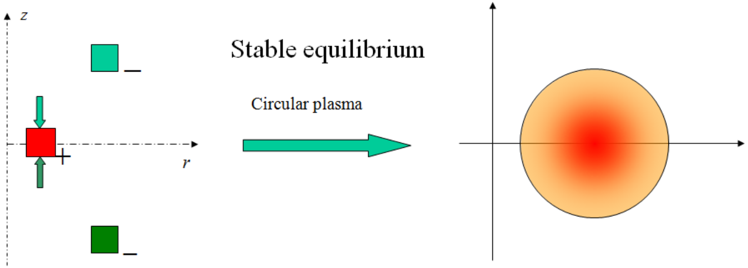
Consider the simplified electromechanical model with three conductive rings, two rings are kept fixed and in symmetric position with respect to the r axis, while the third can freely move vertically.



If the currents in the two fixed rings are equal, the vertical position $z = 0$ is an equilibrium point for the system.

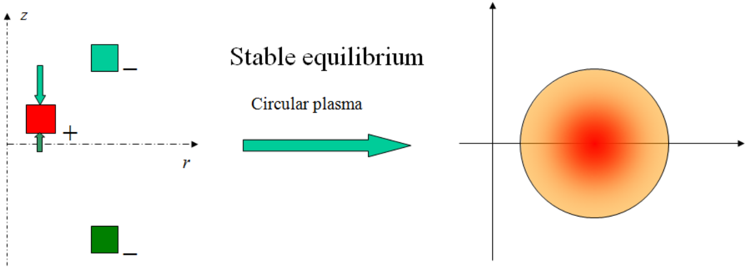
Stable equilibrium - 1

If $\mathrm{sgn}(I_p) \neq \mathrm{sgn}(I)$



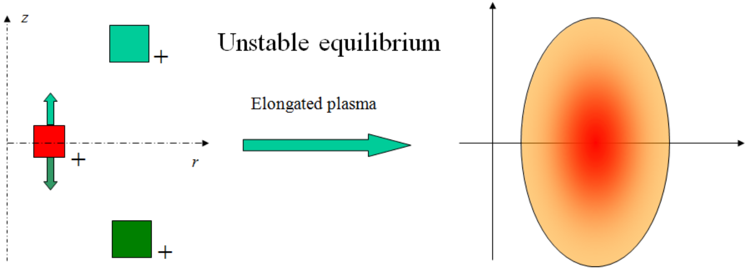
Stable equilibrium - 2

If $\text{sgn}(I_p) \neq \text{sgn}(I)$



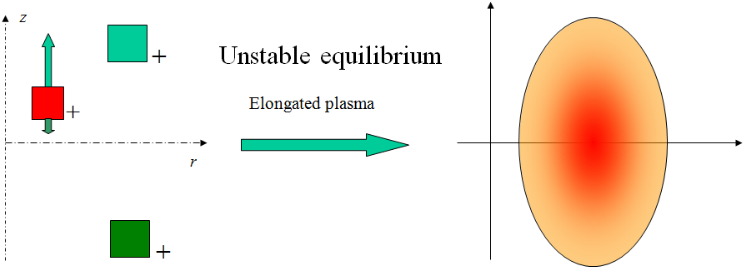
Unstable equilibrium - 1

If $\text{sgn}(I_p) = \text{sgn}(I)$



Unstable equilibrium - 2

If $\text{sgn}(I_p) = \text{sgn}(I)$

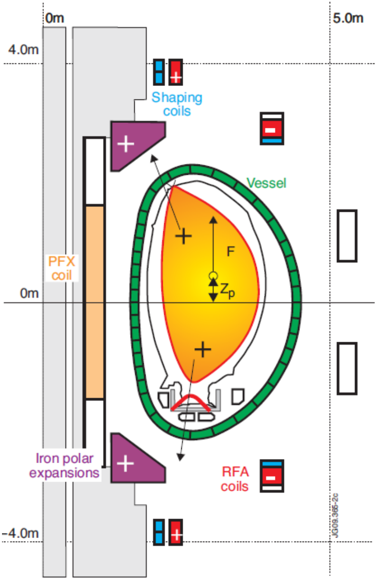


Plasma vertical instability

- The plasma vertical instability reveals itself in the linearized model, by the presence of an unstable eigenvalue in the dynamic system matrix
- The vertical instability growth time is slowed down by the presence of the conducting structure surrounding the plasma
- This allows to use a feedback control system to stabilize the plasma equilibrium, using for example a pair of dedicated coils
- This feedback loop usually acts on a faster time-scale than the plasma shape control loop

Vertical Stabilization system

- single/multiple actuators (RFA @ JET, VS1,VS2, in-vessel coils @ ITER)
- drive the voltages into the actuators
- vertical position plasma stabilization + control of current into the actuators

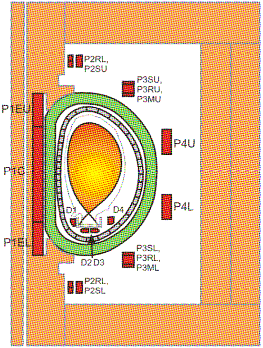


Plasma Shape Control

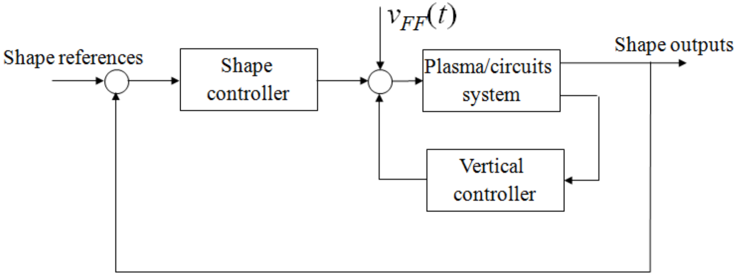
- The problem of controlling the plasma shape is probably the most understood and mature of all the control problems in a tokamak
- The actuators are the Poloidal Field coils, that produce the magnetic field acting on the plasma
- The controlled variables are a finite number of geometrical descriptors chosen to describe the plasma shape

Objectives

- Precise control of plasma boundary
- Counteract the effect of disturbances (β_p and l_i variations)
- Manage saturation of the actuators (currents in the PF coils)

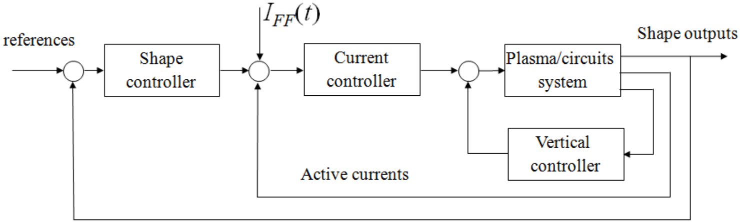


Control scheme - 1



- $v_{FF}(t)$ are the *scenario* voltages feeded in feed-forward to the plant
- Both the VS and the SC generate input voltage variations.

Control scheme - 2



- The scenario is usually specified in terms of feed-forward currents $I_{FF}(t)$.
- It is convenient that the SC generates current references
- A PF currents controller must be designed

It is important to note that plasma shape control and vertical stabilization can be performed on different time scales.

Examples

ITER the time constant of the unstable mode in the ITER tokamak is about 100 *ms*, the settling time of the SC can vary between 15 and 25 *s*.

JET the time constant of the unstable mode in the JET tokamak is about 2 *ms*, the settling time of the SC is about 0.7 *s*.

Plasma current control

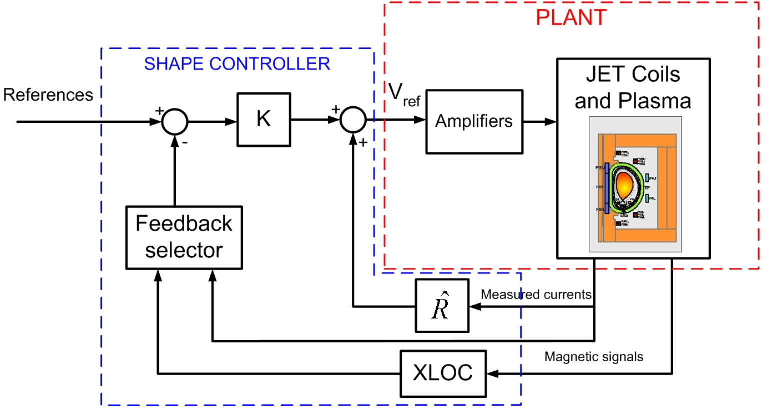
- Plasma current can be controlled by using the current in the PF coils
- Since there is a sharing of the actuators, the problem of tracking the plasma current is often considered simultaneously with the shape control problem
- The PF coils have to generate a magnetic flux in order to drive ohmic current into the plasma
- Shape control and plasma current control are compatible, since it is possible to show that generating flux that is spatially uniform across the plasma (but with a desired temporal behavior) can be used to drive the current without affecting the plasma shape.

SC and VS a JET

At the JET tokamak:

- Two different shape controllers are available
  - the *standard* Shape Controller (SC). This controller can be set in *full current control mode* (acting as a PF currents controler)
  - the eXtreme Shape Controller (XSC)
- The vertical stabilization controller, whose gains are *adaptively* changed during the discharge

JET Shape Controller - Controller Scheme



JET Shape Controller Design

Plasmaless model

$$\mathbf{V}_{PF} = \begin{bmatrix} L_1 & M_{12} & \dots & M_{1N} \\ M_{12} & L_2 & \dots & M_{2N} \\ \dots & \dots & \dots & \dots \\ M_{1N} & M_{2N} & \dots & L_N \end{bmatrix} \frac{\mathrm{d}\mathbf{I}_{PF}}{\mathrm{dt}} + \begin{bmatrix} R_1 & 0 & \dots & 0 \\ 0 & R_2 & 0 & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & R_N \end{bmatrix} \mathbf{I}_{PF}$$

Resistive compensation

$$\mathbf{V}_{PF_{ref}} = \hat{\mathbf{R}}\mathbf{I}_{PF} + \mathbf{K}(\mathbf{Y}_{ref} - \mathbf{Y})$$

Static relationship between PF coils current and controlled variables

$$\mathbf{Y} = \mathbf{T}\mathbf{I}_{PF}$$

Control Matrix

$$\mathbf{K} = \hat{\mathbf{M}}\mathbf{T}^{-1}\Lambda^{-1} \text{ with } \Lambda \text{ diagonal matrix}$$

Closed-loop system

$$\mathbf{MT}^{-1}\dot{\mathbf{Y}} + \mathbf{RI}_{PF} = \mathbf{MT}^{-1}\Lambda^{-1}(\mathbf{Y}_{ref} - \mathbf{Y}) + \mathbf{RI}_{PF} \Rightarrow$$
$$\Rightarrow \ \dot{\mathbf{Y}} = \Lambda^{-1}(\mathbf{Y}_{ref} - \mathbf{Y})$$

By a proper choice of the **T** matrix it is possible to achieve:

- current control mode
- plasma current control mode
- gap control mode

F. Sartori, G. De Tommasi, F. Piccolo
The Joint European Torus
IEEE Control Systems Magazine, April 2006

JET Shape Controller

- Each circuit is used to control a single variable (current, gap, flux)
- Up to 9 different variables can be controlled
- Since plasma current is always controlled, up to 8 gaps can be controlled

XSC "philosophy"

- To control the plasma shape in JET, in principle 8 *knobs* are available, namely the currents in the PF circuits except $P1$ which is used only to control the plasma current
- As a matter of fact, these 8 knobs do not practically guarantee 8 degrees of freedom to change the plasma shape
- Indeed there are 2 or 3 current combinations that cause small effects on the shape (depending on the considered equilibrium).
- **The design of the XSC is model-based. Different controller gains must be designed for each different plasma equilibrium, in order to achieve the desired performances**

SC in current control mode

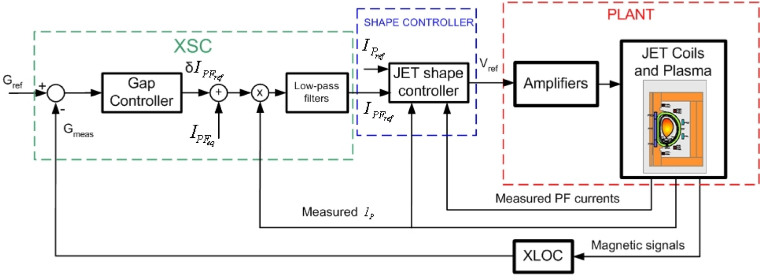
The XSC exploits the standard JET Shape Controller architecture. In particular it sets:

- the P1 circuit in *plasma current control mode*
- the other 8 PF circuits in *current control mode*

Model of the current controlled plant

$$\delta \mathbf{g}(s) = \frac{\widetilde{\mathbf{C}}}{1 + s\tau} \cdot \frac{\delta \mathbf{I}_{PF_{REF}}(s)}{I_P}$$

XSC - Controller scheme



eXtreme Shape Controller (XSC)

- The *eXtreme Shape Controller* (XSC) controls the whole plasma shape, specified as a set of 32 geometrical descriptors, calculating the PF coil current references.
- Let $\mathbf{I}_{PF_N}(t)$ be the PF currents normalized to the equilibrium plasma current, it is

$$\delta\mathbf{g}(t) = \mathbf{C}\ \delta\mathbf{I}_{PF_N}(t).$$

  It follows that the plasma boundary descriptors have the same dynamic response of the PF currents.
- The XSC design has been based on the $\mathbf{C}$ matrix. Since the number of independent control variables is less than the number of outputs to regulate, it is not possible to track a generic set of references with zero steady-state error.

$$\delta\mathbf{I}_{PF_{N_{req}}} = \mathbf{C}^{\dagger}\delta\mathbf{g}_{error}$$

eXtreme Shape Controller (XSC)

- The XSC has then been implemented introducing weight matrices both for the geometrical descriptors and for the PF coil currents.
- The determination of the controller gains is based on the Singular Value Decomposition (SVD) of the following weighted output matrix:

$$\widetilde{\mathbf{C}} = \widetilde{\mathbf{Q}}\,\mathbf{C}\,\widetilde{\mathbf{R}}^{-1} = \widetilde{\mathbf{U}}\,\widetilde{\mathbf{S}}\,\widetilde{\mathbf{V}}^{T},$$

  where $\widetilde{\mathbf{Q}}$ and $\widetilde{\mathbf{R}}$ are two diagonal matrices.
- The XSC minimizes the cost function

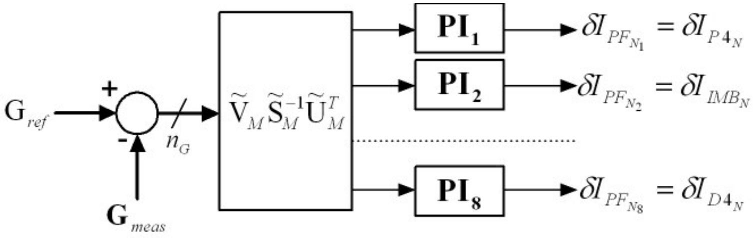
$$\widetilde{J}_1 = \lim_{t\to+\infty} \left(\delta\mathbf{g}_{ref} - \delta\mathbf{g}(t)\right)^T \widetilde{\mathbf{Q}}^T \widetilde{\mathbf{Q}} \left(\delta\mathbf{g}_{ref} - \delta\mathbf{g}(t)\right),$$

  using $\bar{n} < 8$ degrees of freedom, while the remaining $8 - \bar{n}$ degrees of freedom are exploited to minimize

$$\widetilde{J}_2 = \lim_{t\to+\infty} \delta\mathbf{I}_{PF_N}(t)^T \widetilde{\mathbf{R}}^T \widetilde{\mathbf{R}} \delta\mathbf{I}_{PF_N}(t).$$

  (it contributes to avoid PF current saturations)

XSC - Gap controller



SC vs XSC

SC

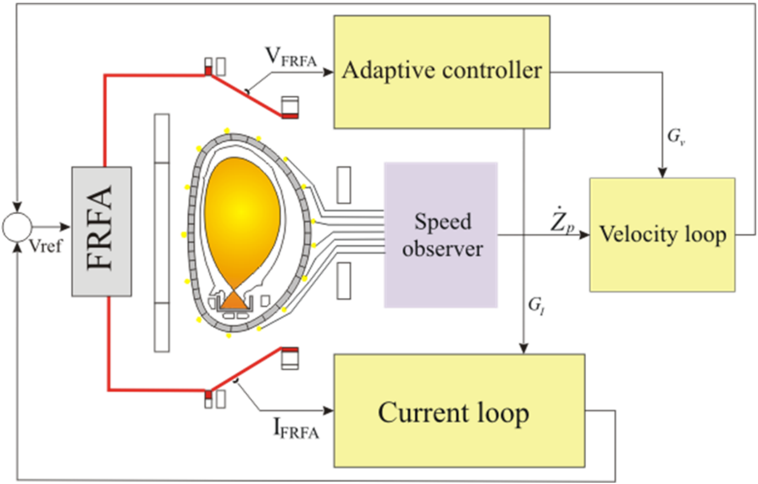
- A few geometric parameters are controlled, usually one gap (Radial Outer Gap, ROG) and two strike points
- The desired shape is achieved precalculating the needed currents and putting these currents as references to the SC
- This gives a good tracking of the references on ROG and on the strike points but the shape cannot be guaranteed precisely
- Shape modifications due to variations of β_p and l_i cannot be counteracted

XSC

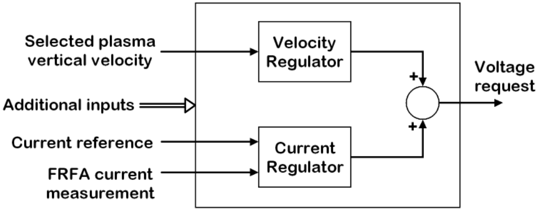
- The shape to be achieved can be chosen
- The XSC receives the errors on 36 descriptors of the plasma shape and calculates the "smallest" currents needed to minimize the error on the "overall" shape
- The controller manages to keep the shape more or less constant even in the presence of large variations of β_p and l_i

Experimental results

JET VS Control Scheme



Simplified control law



- proportional action on plasma velocity $\dot{z}_p$
- proportional-integral action on the current in the actuator

$$V_{req}(s) = G_v s Z_p(s) + G_I\left(1 + \frac{1}{T_I s}\right)\left(I_{ref}(s) - I(s)\right)$$

where typically $I_{ref}(s) = 0$.

Adaptive gains

G_v and G_I are *adapted* during the discharge taking into account the power supply switching frequency, its temperature, the value of the current in the actuator,. . .

VS & JET PCU Project - 1

The **Plasma Control Upgrade (PCU)** project has increased the capabilities of the **JET VS** system so as to meet the requirements for future operations at JET (ITER-like wall, tritium campaign, . . .).

The PCU project was aimed to enhance the ability of the VS system to recover from large ELMs, specially in the case of plasmas with large *growth rate*.

This is especially true for future operation at JET with the beryllium ITER-like wall.

VS & JET PCU Project - 2

G. De Tommasi

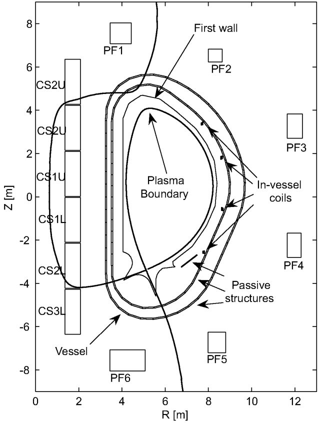
Within the PCU project, the design of the new VS system has included

1. the design of the new power supply for the RFA circuit
2. the assessment of the best choice for the number of turns for the coils of the RFA circuit
3. **the design of the new VS software, so as to deliver to the operator an high flexible architecture**

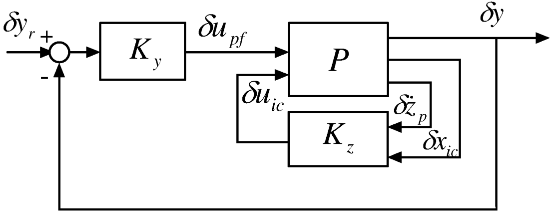
Plasma position and shape control in the ITER using in-vessel coils

Motivations

- During the *design review phase*, it turned out that the high-elongated and unstable plasmas needed for ITER operations can hardly be stabilized using the superconducting PF coils placed outside the tokamak vessel.
- It has been proposed to investigate the possibility of using in-vessel coils to improve the best achievable performance of the VS system.



Control Scheme



Two control loops are designed:

1. the VS system, which stabilizes the plasma vertical position;
2. the plasma current and shape control system, which drives the plasma current error to zero and minimizes the error between the actual plasma boundary and the desired shape reference.

Vertical Stabilization System

Vertical stabilization controller with a simple structure is proposed, in order to envisage effective adaptive algorithms
Let the in-vessel $\delta u_{ic}(t)$ voltage be equal to

$$\delta U_{ic}(t) = k_D \delta \dot{z}_p(t) + k_I \delta I_{ic}(t)$$

Design VS solving a BMI problem

Letting $\mathbf{k}^T = \begin{pmatrix} k_D & k_I \end{pmatrix}$, the two gains k_D and k_I can be chosen so as to fix the closed-loop decay rate in the range $[\theta_{min}, \theta_{max}]$ by solving the following Bilinear Matrix Inequality (BMI)

$$(\mathbf{A} + \mathbf{b}\mathbf{k}^T\mathbf{C})^T\mathbf{P} + \mathbf{P}(\mathbf{A} + \mathbf{b}\mathbf{k}^T\mathbf{C}) < -2\theta\mathbf{P}\,,$$

where $\mathbf{P}$ is a symmetric positive definite matrix and

$$\mathbf{A} = -(\mathbf{L}^*)^{-1}\mathbf{R} \qquad \mathbf{b} = (\mathbf{L}^*)^{-1}\begin{pmatrix} \mathbf{0} \\ 1 \\ \mathbf{0} \end{pmatrix}\,,$$

$$\mathbf{C} = \begin{pmatrix} \mathbf{c}_{1z}^T & c_{2z} & \mathbf{c}_{3z}^T \\ \mathbf{0} & 1 & \mathbf{0} \end{pmatrix}\begin{pmatrix} \mathbf{A} \\ \mathbf{I} \end{pmatrix}\,,$$

with $\theta_{min} < \theta < \theta_{max}$, and $\theta_{min}\,, \theta_{max} > 0$.

The larger θ is, the faster the closed-loop system results.

Plasma current and shape controller

Plasma current and shape control system can act on a slow time scale. It has been shown that the eddy current dynamics can be neglected in the design of the controller K_y. The plant model becomes

$$\delta \dot{\mathbf{I}}_{PF}(t) = (\mathbf{L}^*)^{-1}\delta \mathbf{U}_{PF}(t)\,, \qquad (3a)$$

$$\delta \mathbf{y}(t) = \mathbf{C}\delta \mathbf{I}_{PF}(t)\,. \qquad (3b)$$

The vector $\delta \mathbf{y}(t) = \left(\delta \mathbf{g}^T(t)\ \delta I_p(t)\right)^T$ contains the plasma current plus a set of geometrical descriptors which completely characterize the plasma shape.
The matrix $\mathbf{L}^*$ PF system inductance matrix modified by the presence of the VS loop.

"XSC-like" control law

If $\delta \mathbf{g}(t) = \mathbf{C}_g \delta \mathbf{I}_{PF}(t)$, let us consider the following singular value decomposition

$$\mathbf{C}_g = \mathbf{U}_g \mathbf{\Sigma}_g \mathbf{V}_g^T ,$$

The control law is chosen as

$$\delta \mathbf{U}_{PF}(t) = \mathbf{K}_{SF} \delta \mathbf{I}_{PF}(t) + \mathbf{K}_{P_1} \mathbf{V}_g \mathbf{\Sigma}_g^{-1} \mathbf{U}_g^T \delta \mathbf{g}(t)$$
$$+ \mathbf{K}_{I_1} \mathbf{V}_g \mathbf{\Sigma}_g^{-1} \mathbf{U}_g^T \int_0^t \left(\delta \mathbf{g}(t) - \delta \mathbf{g}_r(t) \right) dt + k_{P_2} \delta I_p(t) + k_{I_2} \int_0^t \left(\delta I_p(t) - \delta I_{p_r}(t) \right) dt ,$$

where $\delta \mathbf{g}_r(t)$ and $\delta I_{p_r}(t)$ are the reference on the plasma geometrical descriptors and the plasma current.

Simulation results - 1

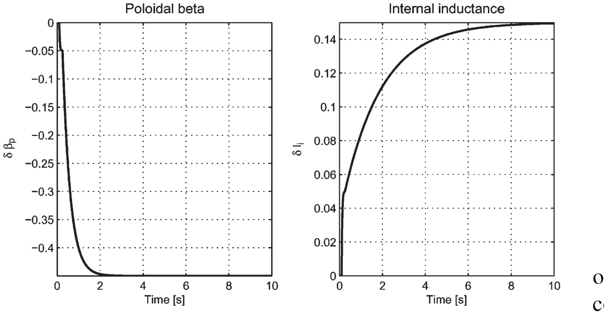


Figure: Analysis of the performance during an H-L transition. Time traces of $\delta\beta_p(t)$ and $\delta l_i(t)$.

Simulation results - 2

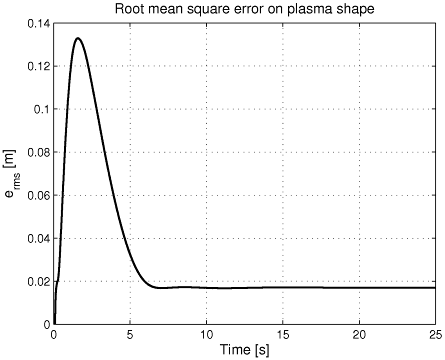


Figure: Analysis of the performance during an H-L transition. Mean square error on the controller plasma shape descriptors.

Simulation results - 3

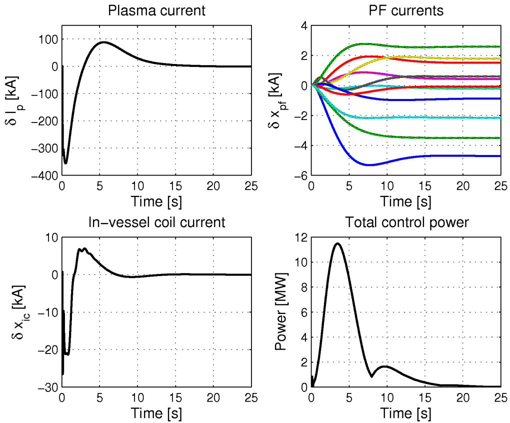


Figure: Analysis of the performance during an H-L transition. This figure shows the time traces of the plasma current variation $\delta I_p(t)$, of the control currents $\delta \mathbf{x}_{pf}(t)$, $\delta x_{ic}(t)$,and the total power required to track the desired shape reference.

Conclusions

An overview of the three basic magnetic control problems has been given:

- Vertical Stabilization
- Shape Control
- Plasma Current Control

. . .let's do practice in the lab!

THE END

Thank you!

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